Non-orientable regular maps of Euler characteristic equal to the negative of an odd prime power

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OU and STU

Joint work with M. Conder, N. Gill and I. Short

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Regular maps - informally

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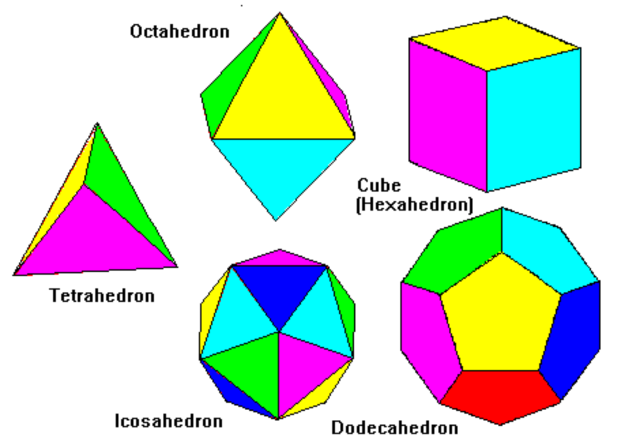

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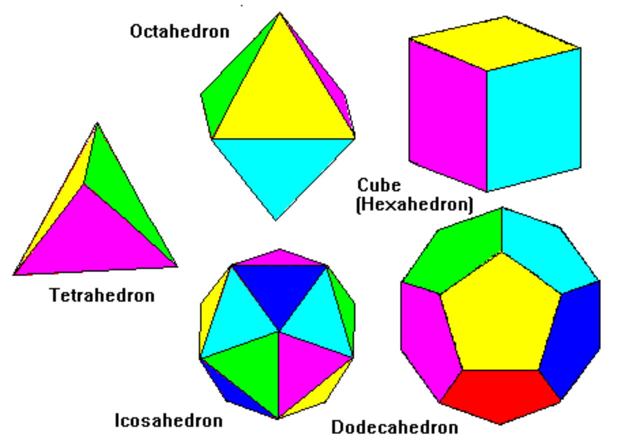

Regular maps are generalisations of Platonic maps to arbitrary surfaces.

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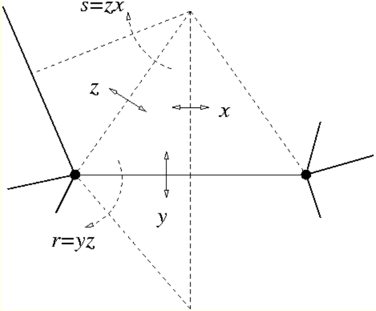

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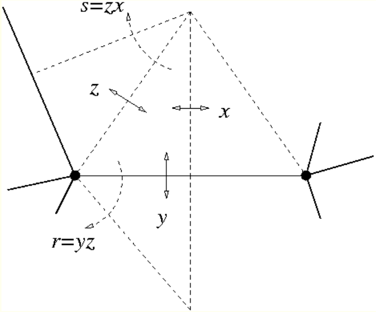

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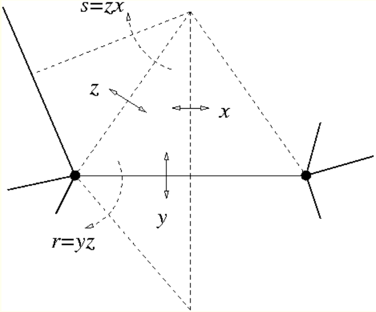

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Conversely, every group with such a presentation determines a regular map.

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A computer-assisted classification for $\chi \geq -28$ [Conder, Dobcsányi 2001]

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Classification for 'small' genera carried over to $\chi \geq -600$ with the help of more powerful computational methods [Conder 2013].

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More than 3/4 of values of χ are non-gaps [Conder, Everitt 1995].

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Moreover, if $\chi = -r^\ell$ for some odd prime r, then G is isomorphic to $PSL(2,q)$ or $PGL(2,q)$.

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Theorem 3. *Let G be the automorphism group of a regular map of type (k, m) with $\chi = -r^\ell$ for an odd prime r. If $G/O \simeq PSL(2, q)$ for some prime power q, then one of the following cases occur:*

r	q	(k, m)
3	5	$(5, 5)$
3	5	$(3, 15)$
7	13	(3,13)
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Note: In each case there exist infinitely many examples (by Theorem 2).

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(3) $r = p$, $t = 1$ *and* $\{k', m'\} = \{q-1, q+1\}$;

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THANK YOU.